Integration der e-Funktion
Übung

1. Geben Sie eine Stammfunktion zu f an.

a) $f(x) = e^{2x-1}$

b) $f(x) = e^{\frac{1}{4}x+5}$

c) $f(x) = e^{4-x}$

d) $f(x) = x + e^{x}$

e) $f(x) = \frac{1}{3e^{x}}$

f) $f(x) = \frac{1}{e^{3x-2}}$

g) $f(x) = 2 - 3e^{1-3x}$

h) $f(x) = e^{x-1} - e^{-x+1}$

i) $f(x) = x^{-2} + e^{-2x+1}$

2. Schreiben Sie integralfrei!

a) $\int e^{x}\,dx$

b) $\int e^{2x-3}\,dx$

c) $\int \frac{1}{e^{x-1}}\,dx$

d) $\int e^{0,5x+1} + 2\,dx$

e) $\int e^{2x} - e^{-2x} dx$

f) $\int 5e^{x-1} \cdot e^{x+1}\,dx$

3. Berechnen Sie den Wert des bestimmten Integrals.

a) $\int_{-1}^{1} e^{x}\,dx$

b) $\int_{-2}^{1} e^{3x-2}\,dx$

c) $\int_{0}^{3} 3e^{2x}\,dx$

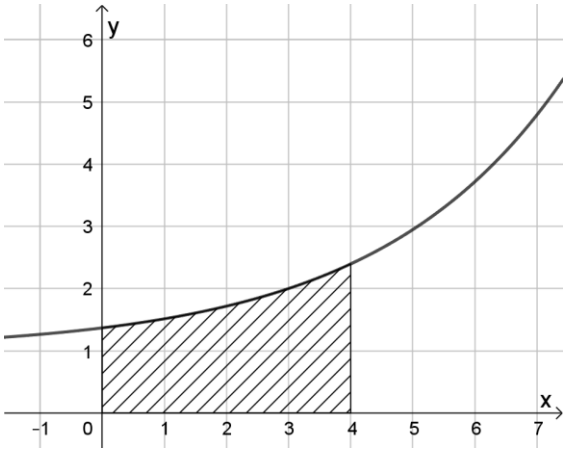
d) $\int_{0}^{1} e^{\frac{1}{3}x}\,dx$

e) $\int_{\ln(2)}^{2} (1 - e^{x})^{2}\,dx$

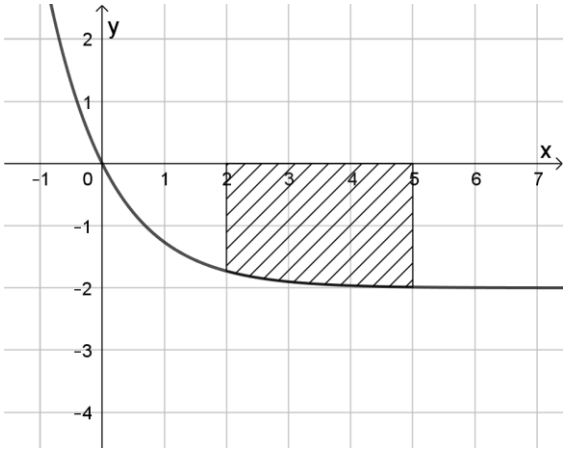
f) $\int_{0}^{1} (e^{x} - e^{-x})^{2}\,dx$

4. Berechnen Sie die Maßzahl der gekennzeichneten Fläche.

a) $f(x) = e^{\frac{1}{3}x-1} + 1$



b) $f(x) = 2e^{-x} - 2$



Integration der e-Funktion
Lösung

1.

a) Z.B. $F(x) = \frac{1}{2}e^{2x-1}$

b) Z.B. $F(x) = 4e^{\frac{1}{4}x+5}$

c) Z.B. $F(x) = -e^{4-x}$

d) Z.B. $F(x) = \frac{1}{2}x^2 + e^x$

e) Z.B. $F(x) = -\frac{1}{3}e^{-x} = -\frac{1}{3e^x}$

f) Z.B. $F(x) = -\frac{1}{3e^{3x-2}}$

g) Z.B. $F(x) = 2x + e^{1-3x}$

h) Z.B. $F(x) = e^{x-1} + e^{-x+1}$

i) Beispielsweise $F(x) = -x^{-1} - \frac{1}{2}e^{-2x+1}$

2.

a) $e^x + c$

b) $\frac{1}{2}e^{2x-3} + c$

c) $-e^{-x+1} + c = \frac{-1}{e^{x-1}} + c$

d) $2e^{0,5x+1} + 2x + c$

e) $\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x} + c$

f) $5e^{x-1} \cdot e^{x+1} = 5e^{2x}$, also $\frac{5}{2}e^{2x} + c$

3.

a) $\int_{-1}^{1} e^x\,dx = [e^x]_1^1 = e^1 - e^{-1} = e - \frac{1}{e} \approx 2,35$

b) $\int_{-2}^{1} e^{3x-2}\,dx = \left[\frac{1}{3}e^{3x-2}\right]_2^1 = \frac{1}{3}e^1 - \frac{1}{3}e^{-8} \approx 0,91$

c) $\int_0^3 3e^{2x}\,dx = \left[\frac{3}{2}e^{3x-2}\right]_0^3 = \frac{3}{2}e^6 - \frac{3}{2} \approx 605,64$

d) $\int_0^1 e^{\frac{1}{3}x}\,dx = \left[3e^{\frac{1}{3}x}\right]_0^1 = 3e^{\frac{1}{3}} - 3 \approx 1,19$

e) $\int_{\ln(2)}^{2}(1-e^x)^2\,dx = \int_{\ln(2)}^{2} 1 - 2e^x + e^{2x}\,dx = \left[x - 2e^x + \frac{1}{2}e^{2x}\right]_{\ln(2)}^{2}$
$= 8 - 2e^2 - \frac{1}{2}e^4 - \ln(2) \approx -34,77$

f) $\int_0^1 (e^x - e^{-x})^2\,dx = \int_0^1 e^{2x} - 2 + e^{-2x}\,dx = \left[\frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x}\right]_0^1$
$= \frac{1}{2}e^2 - 2 - \frac{1}{2}e^{-2} \approx 1,63$

4.

a) $A = \int_0^4 e^{\frac{1}{3}x-1} + 1\,dx = \left[3e^{\frac{1}{3}x-1} + x\right]_0^4 = 3e^{\frac{1}{3}} + 4 - 3e^{-1} \approx 7,08\ \text{FE}$

b) $A = -\int_2^5 2e^{-x} - 2\,dx = -[-2e^{-x} - 2x]_2^5 = 2e^{-5} + 6 - 2e^{-2} \approx 5,74\ \text{FE}$